Analogical Problem Solving

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INTRODUCTION.

Where *do* new ideas come from? What psychological mechanisms underlie creative insight? This fundamental issue in the study of thought has received a great deal of informal discussion, but little empirical psychological investigation. The anecdotal reports of creative scientists and mathematicians suggest that the development of a new theory frequently depends on noticing and applying an analogy drawn from a different domain of knowledge. ... The hydraulic model of the blood circulation system, the planetary model of atomic structure, and the "billiard ball" model of gases all represent major scientific theories founded on analogies.

While the process of solving analogy test items of the form A:B::C:D has been studied quite extensively (Sternberg, 1977a, 1977b), there has been little experimental investigation of analogical thinking in more complex problem-solving tasks. Some studies have examined transfer between homomorphic or isomorphic versions of puzzle problems, such as the "missionaries and cannibals" (Reed, Ernst, & Banerji, 1974) and Tower of Hanoi (Hayes & Simon, 1977) puzzles. These are relatively "well-defined" problems ... in which the initial conditions, legal operations, and goal state are explicitly specified. In contrast, anecdotal reports of the use of analogies typically involve problems that are much less well defined. The present study was designed to investigate the use of analogies between disparate domains as a guide to finding solutions for an ill-defined problem.

THE RADIATION PROBLEM AND ITS ANALOGIES

Our basic experimental procedure was to provide subjects with a story analogy, describing a problem and its solution, and then to observe how subjects used the analogy in solving a subsequent target problem. The target problem was Duncker's (1945) "radiation problem," which in our experiments was stated as follows.

Suppose you are a doctor faced with a patient who has a malignant tumor in his stomach. It is impossible to operate on the patient, but unless the tumor is destroyed the patient will die. There is a kind of ray that can be used to destroy the tumor. If the rays reach the tumor all at once at a sufficiently high intensity, the tumor will be destroyed. Unfortunately, at this intensity the healthy tissue that the rays pass through on the way to the tumor will also be destroyed. At lower intensities the rays are harmless to healthy tissue, but they will not affect the tumor either. What type of procedure might be used to destroy the tumor with the rays, and at the same time avoid destroying the healthy tissue?

There are several reasons why the radiation problem seemed especially suitable for use in a study of analogical problem solving. First, it has all the hallmarks of the kind of "ill-defined" problem for which an analogy from a remote domain might trigger a creative insight. The desired goal state is specified only at a relatively abstract level, and the permissible operations that might be used to achieve the goal are left very open ended. As a consequence, the possible solution proposals vary considerably. This made it possible to test for the use of analogies by attempting to influence the specific solutions that subjects would generate.

In addition, we were able to benefit from Duncker's analyses of the performance of subjects who worked on the problem without receiving an analogy. Duncker identified three broad categories of proposed solutions to the radiation problem: (1) reducing the intensity of the rays as they pass through the healthy tissue; (2) avoiding contact between the rays and healthy tissue; and (3) altering the relative sentitivity to rays of the healthy tissue and the tumor (e.g., by immunizing the healthy tissue or sensitizing the tumor). Our analogies were designed to guide subjects toward specific versions of the first two classes of proposals.

Our general aim in the present study, then, was to explore the process by which subjects use analogies between remote domains to generate problem solutions. Consequently, we wrote a series of stories far removed from the medical domain, each involving a military problem and its solution, which were analogous to the radiation problem.

A FRAMEWORK FOR ANALOGICAL PROBLEM SOLVING

It is important to develop a general conceptual framework within which specific issues concerning the role of analogies in problem solving can be formulated. What is meant by analogy, and how can an analogy be used to generate a problem solution?

In order to make our discussion more concrete we will consider the major story analogy used in the present experiments and the corresponding solution to the radiation problem. In the Attack-Dispersion story, a general wishes to capture a fortress located in the center of a country. There are many roads radiating outward from the fortress. All have been mined so that while small groups of men can pass over the roads safely, any large force will detonate the mines. A full-scale direct attack is therefore impossible. The general's solution is to divide his army into small groups, send each group to the head of a different road, and have the groups converge simultaneously on the fortress. The analogous solution to the radiation problem is to simultaneously direct multiple low-intensity rays toward the tumor from different directions. In this way the healthy tissue will be left unharmed, but the effects of the multiple low-intensity rays will summate and destroy the tumor.

At an intuitive level the parallels between the Attack-Dispersion story and the radiation problem are clear. Both situations involve an object that must be overcome, surrounded by objects that must be preserved. The target object in each case occupies a topographically central position in its environment. In each situation the protagonist has available a weapon with an effect proportional to the intensity or amount of the weapon that is used, and so on.

How might people represent these analogical relationships and use them to generate a solution to the target problem? This is not an easy question to answer. First of all, both the story and the problem must be read and understood. In attempting to describe this type of analogical problem solving we thus inherit all the problems associated with text comprehension. In particular, perception of analogy hinges on semantic knowledge and inference procedures. Since no general theory of language understanding is available, we must of necessity gloss over many important issues related to the understanding process. However, .. .there appear to be close ties between the concept of analogy and the concept of "schema," which has been widely applied in discussions of prose comprehension. In essence, both an analogy and a schema consist of an organized system of relations. Consequently, the framework for analogical problem solving presented here will draw its conceptual vocabulary from various schema-based models, as well as from Sternberg's (1977a, 1977b) model of component processes involved in analogical reasoning. We will first consider how analogy might be represented, and then how this representation could be used to generate a solution to a problem.

The Representation of Analogy

A system of representation for analogy must be able to describe a fundamental property of such relational systems, namely, that analogy may be defined at multiple levels of abstraction. For example, at a relatively low level of abstraction the Attack-Dispersion story and the radiation problem have a variety of corresponding details (e.g., small groups of soldiers correspond to low-intensity rays). At a more abstract level, the story and the problem both involve the goal of overpowering an object located in a region that must be preserved.

The multileveled nature of analogy can perhaps be understood in the context of Kintsch and Van Dijk's (1978) theory of prose representation. They argue that the understanding process may involve the iterative application of a set of inference rules that generate increasingly abstract "macrostructure" representations of a prose passage. These macrostructures essentially correspond to summaries of the passage at various levels of generality. In the case of a problem-oriented story such as the Attack-Dispersion story, an abstract level of macrostructure might state a general solution principle (e.g., to destroy a target when direct application of a large force is harmful to the surrounding area, disperse the attacking forces, and have them converge at the target). The process of extracting a solution principle might thus be viewed as a special case of the process of deriving macrostructures for a body of information. While much remains to be learned about how this process operates, the three specific inference rules proposed by Kintsch and Van Dijk (which they term "deletion," "generalization," and "construction") would seem readily applicable to the type of story analogies we are considering here.

Kintsch and Van Dijk emphasize that control processes are required to select a level of macrostructure analysis consistent with the person's processing goals. Similarly, we assume there is an optimal level of abstraction at which analogical relations may be represented in order to effectively guide the solution process. Indeed, an important empirical issue is to determine what factors influence this optimal level of abstraction.

We will now consider in more detail how an analogy between two relational systems might be represented, assuming an appropriate level of macrostructure has been derived. To pursue our example, Table [13.1] presents our own summary of the Attack-Dispersion story, as well as a summary, of the radiation problem and its dispersion solution. These summaries are intended to reflect the major causal connections within both the story and the problem, and to illustrate the major analogical relations between them. The sentences in Table [13.1] are numbered to correspond to an approximate propositional analysis presented in Fig. j 13.1]. Propositions from the story and from the radiation problem are matched to indicate analogical relations, and the propositions corresponding to the dispersion solution to the radiation problem (which a person would be required to generate) are italicized in both the table and the figure. Note that some of the propositions included in the story summary (e.g., proposition 11) are inferences, which are not directly stated in the original story. ...

The notation in Fig. [13.1] consists of propositional functions, in which predicates are followed by one or more arguments (enclosed in parentheses). The arguments fill various semantic roles, such as agent, object, and location. Propositions may themselves serve as arguments, so in

TABLE 13.1
A Summary of Attack-Dispersion Story and of Corresponding Solution to Radiation Problem (See Fig. 13.1)

Proposition number	
	Attack-Dispersion story
1-2	A fortress was located in the center of the country.
2a	Many roads radiated out from the fortress.
3-4	A general wanted to capture the fortress with his army.
5-7	The general wanted to prevent mines on the roads from destroying his army and neighboring villages.
8	As a result the entire army could not attack the fortress along one road.
9-10	However, the entire army was needed to capture the fortress.
11	So an attack by one small group would not succeed.
12	The general therefore divided his army into several small groups.
13	He positioned the small groups at the heads of different roads.
14-15	The small groups simultaneously converged on the fortress.
16	In this way the army captured the fortress.
	Radiation problem and dispersion solution"
Γ-2'	A tumor was located in the interior of a patient's body.
3'—4'	A doctor wanted to destroy the tumor with rays.
5' -7'	The doctor wanted to prevent the rays from destroying healthy tissue.
8'	As α result the high-intensity rays could not be applied to the tumor along one path.
9'-10'	However, high-intensity rays were needed to destroy the tumor.
11'	So applying one low-intensity ray would not succeed.
12'	*The doctor therefore divided the rays into several low-inlensilv rays.*
13'	*Hepositioned ihelow-intensityraysatmultiplelocationsaroundthepatienl'sbody.*
14'-15'	*The low-intensity rays simultaneously converged on the tumor.*
16'	*In this way the rays destroyed the tumor.*

" Italicized propositions summarise the target dispersion solution.

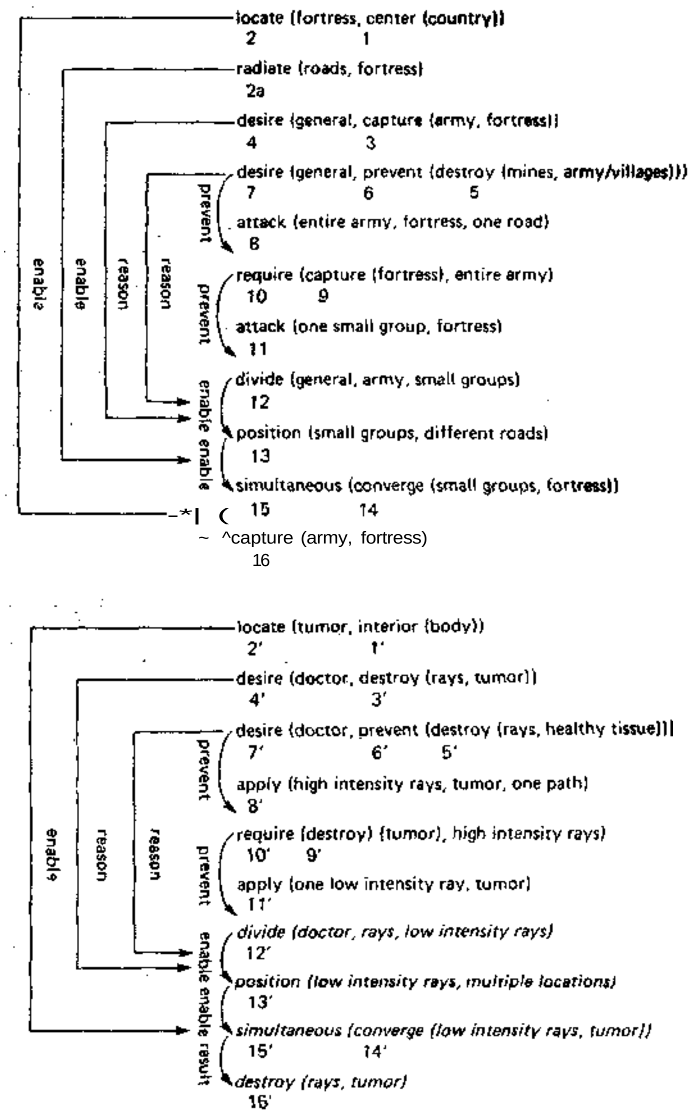

FIG. 13.1 Analogical correspondences between the Attack-Dispersion story and the radiation problem.

several cases one proposition is embedded in another. We make no claims about the logical adequacy or completeness of the representation in Fig. [13.1]; indeed, many of the indicated arguments (e.g., "low-intensity rays") clearly could be further decomposed. However, separation of relations (predicates) and arguments serves to highlight the critical properties of analogy between remote domains: similarity of corresponding relations despite dissimilarity of corresponding arguments.

The notation in Fig. [13.1] has been augmented by labeled arcs that represent the major causal connections within the Attack-Dispersion story and the radiation problem. The labels are inspired by but not identical to the analysis of causa! types offered by Schank and Abelson (1977). ... Roughly, a goal can be a "reason" for an action; a state or action can "enable" a subsequent action; an action can "result" in a subsequent state; and a state or action can "prevent" a subsequent action. Again, the adequacy of this analysis is not really critical for our present purpose, which is simply to make salient the correspondences in causal structure between the story and the problem, particularly with respect to the target solution. Note that the labeled arcs are equivalent to higher-order predicates that embed numbered propositions as arguments.

The representation in Fig. [13.1] can be used to highlight a variety of properties of analogy, as well as some of the issues we have glossed over· in constructing this representation. Fundamentally, an analogy consists of a mapping between two sets of propositions. Propositions are matched on the basis of similarity between the corresponding relations. Note that similarity need not imply identity. There need only be a consistent transformation that maps one set of relations onto the other. Boden (1977) gives the example of the Black Mass, which is based on systematic semantic reversals of the Catholic ritual. In the case of our story and problem, we wrote the summaries to maximize the similarity of the relations. Yet dividing an army (proposition 12), for example, is clearly not quite the same as dividing high-intensity rays (proposition 12')· To generate this aspect of the parallel solution the person would need to take account of the relevant differences between an army and rays, perhaps explicitly introducing multiple machines that each emit low-intensity rays. In order to map proposition 1 onto Γ a person must have semantic knowledge of the relation between the meanings of "center" and "interior." Incidentally, note that we assume proposition Γ is included in the macrostructure for the problem as an inference based on knowledge of where the stomach is located. In the case of propositions 16 and 16', we assume the person will use semantic knowledge to transform the relation of "capturing" into the relation of "destroying," operating on the common semantic core (i.e., "overcoming") that links the two relations.

As Fig. [13.1] indicates, there is clearly a high degree of correspondence between the propositions of the story and of the problem. At the same time, the systems are not perfectly isomorphic. It is probably the case that analogies used to guide problem solving are generally incomplete in some respects. ... For example, proposition 2a in the Attack-Dispersion story, which states that many roads radiate outward from the fortress, has no parallel in the statement of the radiation problem. Note that in the story this proposition serves as an important enabling condition for the solution. The absence of any explicit mention in the radiation problem of multiple paths to the target object would presumably hinder generation of the dispersion solution.

For the above example it is plausible to argue that people must infer the fact that there are multiple potential "routes" to the tumor in the course of generating the dispersion solution, even though no such inference is represented in Fig. [13.1]. But in addition, Fig. [13.1] reveals at least one clear disanalogy between the story and the problem. In a complete analogy, there is a consistent mapping between pairs of arguments. That is, wherever argument A occurs in one relational system, argument A' occurs in the other. For example, in Fig. [13.1] the role the fortress plays in the story consistently maps onto the role the tumor plays in the problem. Note that the role of the army usually corresponds to that of the rays. However, this is not the case in propositions 5 and 5'. In the Attack-Dispersion story, sending the entire army down one road will result in destruction of the *army* (as well as neighboring villages) by *mines'*, whereas in the radiation problem applying high-intensity rays to the tumor will result in destruction of the *healthy tissue* by the *rays.* In other words, the army and the rays do not fill corresponding semantic roles in propositions 5 and 5': rather, the army is the *object* of the process of destruction in 5, while the rays are the *instrument* of the destruction in 5'.

This example illustrates that degree of analogy in part depends on the level of abstraction at which the analogy is defined. In macrostructures slightly more abstract than those depicted in Fig. [13.1], the fact that an attack by the entire army is impossible would map onto the fact that direct application of high-intensity rays is impossible. At this level the roles of the army and of the rays correspond appropriately. However, the story and the problem are disanalogous at the more specific level depicted in Fig. [13.1] (i.e., the level of the *reasons* why the two respective courses of action are blocked). This observation suggests that for use in solving a problem the optimal level of abstraction tor representing an analogy may be that which maximizes the degree of correspondence between the two relational systems. In many cases a very detailed representation will include disanalogous relations, while a very abstract representation will omit information about important correspondences.

The Process of Analogical Problem Solving

So far we have been discussing how analogical relations may be represented: we must now consider how this information might be used to generate a solution to a problem. For our story analogy and target problem the solution process appears to require three major steps.

1. A representation of the story analogy and of the target problem (i.e., its initial state and goal state) must be constructed, as described above.
2. The representation of the story must be mapped onto that of the problem. If the story and the problem are drawn from remote domains, as in our example, the correspondences between arguments will not be immediately obvious. We would therefore expect the mapping process to be initiated by detection of similar relations in the two systems. For example, the person might notice that propositions 2 and 2' both involve location. Accordingly, a mapping between the two propositions will be established. This will automatically establish a mapping between the corresponding arguments (i.e., the fortress and the tumor, the center of the country and the interior of the body). Once a few such correspondences have been detected, the mapping process may proceed in a more "top-down" manner, guided by expectations that previously mapped arguments will continue to play parallel roles. For example, having mapped propositions 2 and 2', the person might assume that 8 maps onto 8' because the role of the fortress should correspond to that of the tumor.
3. Finally, the person must use the mapping to generate the parallel solution to the target problem. This can be done by constructing a set of solution propositions for the target problem that correspond to the solution propositions of the story. For example, consider how proposition 12' might be generated on the basis of proposition 12. The mapping process will have identified the general with the doctor and the army with the rays. Accordingly, "doctor" and "rays" will be used to fill the argument slots corresponding to "general" and "army." In addition, the relation between the general and the army in 12 will be used to construct a parallel relation between the doctor and the rays in 12'. Thus the idea of the general dividing the army into several small groups will be transformed into the idea of the doctor dividing the rays into a number of low-intensity rays, initiating the dispersion solution to the radiation problem.

ISSUES AND EXPERIMENTS

A number of important questions arise within the framework of the process model we have outlined. A major issue, which we touched upon earlier, concerns the level of macrostructure at which the mapping process takes place. At one extreme the solution process might amount to abstracting a solution principle from the story and then applying it to the target problem. At the other extreme subjects might map the correspondences between the story and the problem at the most detailed possible level. It is possible, of course, that the mapping process may actually proceed partially in parallel on several different levels.

Even at a single level of macrostructure, there may be strategic variations in the degree of mapping that takes place during the solution process. For example, subjects need not derive the entire set of correspondences outlined in Fig. [13.1] in order to generate the dispersion solution. One possibility, which we will term the "solution-focusing" strategy, is that subjects attempting to apply the story analogy will immediately identify the solution propositions of the story. By doing the minimal amount of mapping required to match the arguments in these propositions with arguments in the radiation problem, the parallel solution could be generated. Subjects using the solution-focusing strategy might thus solve the target problem without entirely grasping the correspondences between the problem statements in the story and in the radiation problem.

Given the lack of empirical research on analogical problem solving, even more basic issues arise. We have sketched a model of how in principle a problem might be solved on the basis of an analogy. However, we do not know whether subjects could actually execute this kind of process for our story analogies and target problem. There seem to be at least three distinct ways in which subjects who have a relevant analogy available might nonetheless fail to derive the parallel solution to a target problem. The first and most basic is that subjects might be unable to successfully apply the story analogy even if they tried. Second, even if a story analogy is potentially useful, subjects might be unable to locate it in memory, especially if it had been encoded in the context of irrelevant distractor stories. Third, subjects might be able to retrieve a potentially useful analogy and yet fail to spontaneously notice its relevance to the target problem.

The experiments reported below were designed to explore these and related issues. I . . .1

EXPERIMENT I

Experiment I was designed to demonstrate that subjects can use an analogy from a remote domain as a hint for solving a problem. Subjects first read a story analogy, and then attempted to propose as many solutions as possible to the radiation problem. By varying the nature of the solution suggested by the story, we hoped to influence the likelihood that subjects would generate specific solutions to the target problem. Subjects' "thinking aloud" protocols were tape recorded and later analyzed as a source of evidence regarding the process of analogical problem solving.

Subjects in three experimental conditions read one of three stories about a military problem and its solution.... Table [13.2] informally illustrates the correspondences among the three stories and the radiation problem. The statement of the radiation problem (see Introduction) was worded so as to minimize obvious lexical or syntactic correspondences with the story analogies. The Attack-dispersion, Open Supply Route, and Tunnel stories all have identical first paragraphs describing the problem setting, desired goal, and the constraints on a solution. These aspects of the stories are analogous to the radiation problem, as discussed earlier (see Fig. [13.1]).

However, the stories differ in their second paragraphs, which state the general's solution to his problem. In the Attack-Dispersion story (described in the Introduction) the general divides his army into small groups and sends them simultaneously down different roads to the fortress. The analogous solution to the radiation problem is the "dispersion" solution: have multiple low-intensity rays converge at'the tumor. This is a very effective solution, but one which subjects seldom generate spontaneously. Duncker (1945) reported that only 2 of 42 subjects arrived at this dispersion solution, and both were prompted by the experimenter. A basic difficulty that appears to block generation of this solution is that people do not spontaneously think of rays as having the property of "divisibility." [...]

In the Open Supply Route story the general discovers an unblocked road leading to the fortress, and sends the entire army down this open road. An analogous radiation solution is to direct high-intensity rays down the esophagus (or some other open passage, such as the intestines) to the stomach. This solution was generated relatively frequently by the subjects tested by Duncker (29% gave the open passage solution as opposed to only 5% who gave the dispersion solution). In the Tunnel story the general digs an underground tunnel and sends his army through it to the fortress. Analogous radiation solutions might be to operate to expose the tumor to the rays, or to insert a tube through the stomach wall and send rays through it to the tumor. Many of Duncker's subjects (40%) spontaneously suggested such solutions. However, such procedures to create an open route to the tumor involve operating, and hence conflict with one of the constraints imposed on the radiation problem (that it is impossible to operate). The Tunnel story is therefore a kind of "false analogy" to the radiation problem. That is, although the problem statements are analogous, the solution suggested by the story is inappropriate. If the analogy is

TABLE 13.2
Schematic Outline of Duncker's Radiation Problem Showing Correspondences with Analogous Stories

Problem Statement	*Radiation problem*	*Siory analogies—Experiment I*
Problem setting	Doctor has rays. Patient has tumor. Tumor in stomach, surrounded by healthy tissue.	General has army. Country has dictator. Dictator in fortress in center of country, surrounded by villages. Roads radiate from fortress like spokes on a wheel.
Desired goal	Destroy tumor with rays.	Capture fortress with army.
Problem constraints	High-intensity rays destroy tumor but also destroy healthy tissue.	Entire army can capture fortress, but large group detonates mines on roads, destroying army and villages.
	Low-intensity rays destroy neither tumor nor healthy tissue. Impossible to operate.	Small group of men can pass safely over roads but can not capture fortress.
Solutions		
Type I	Reduce intensity of rays on way to tumor.	Reduce size of group traveling to fortress on one road.
Dispersion (Attack-Dispersion story)	(1) Many low intensity rays (2) From different directions (3) Simultaneously	(1) Many small groups of men (2) From different directions (3) Simultaneously
Type II	Avoid contact between rays and healthy tissue.	Avoid contact between army and mines.
(1) Open passage (Open Supply Route story)	Send high-intensity rays through an open route (e.g., esophagus).	General discovers road that is not mined, and sends entire army down this road.
(2) Operation (Tunnel story)	Make an incision in stomach wall, removing healthy tissue from path of rays, and apply high intensity rays to tumor."	Dig tunnel under mines, and send entire army through.
Resulting goal state	Radiation of high-intensity reaches tumor. Tumor destroyed. Healthy tissue intact.	Entire army reaches fortress. Fortress captured. Army and villages preserved.

' Incision violates constraint.

nevertheless applied, subjects given the Tunnel story might be especially likely to momentarily disregard the problem constraints and propose an operation solution to the radiation problem.

Although the above analysis of the analogous relationships between various solutions to the military problem and to the radiation problem was initially based on the experimenters' intuitions, we will see below that subjects' ratings essentially confirm the validity of this analysis.

The primary prediction in Experiment I was that each story analogy would tend to increase the frequency of the analogous solution to the radiation problem, relative to the solution frequencies obtained for control subjects given no prior story. However, there are additional ways in which the story analogies might influence the solutions given to the target problem. First, note that the problem statements for all three stories contain all the enabling conditions (see Fig. [13.1]) for generating the dispersion solution (e.g., the central location of the fortress, the roads radiating outward, the fact that small groups can travel on the roads). Accordingly, subjects might spontaneously think of the dispersion solution to the general's problem, and then use it to generate the parallel solution to the radiation problem. If so, subjects given the Open Supply Route and Tunnel stories might also produce the dispersion solution more often than would control subjects.

It is also possible that giving subjects a story analogy may actually hinder the generation of nonanalogous solutions. That is, attempting to generate a parallel solution to the target problem may create a kind of "set" effect, so that other possible solutions (e.g., immunizing the healthy tissue to protect it from the rays) will not be discovered. If such a set effect is obtained, control subjects should produce more total solutions than experimental subjects, and in addition there should be qualitative differences between the solutions produced by control subjects versus subjects given story analogies.

[…]

[. . .] Subjects' protocols for the radiation problem were transcribed and scored for the presence of various types of proposed solutions, by two independent scorers. For this purpose any suggestion, even if it was eventually rejected by the subject, was counted as a proposed solution. The results of major interest concern the three types of proposals that are analogous to the solutions embodied in the story analogies—the dispersion solution, the open passage solution, and operation solutions. Table [13.3] presents the percentage of subjects in each condition who produced these various types of proposed solutions. The frequency of each solution was highest for subjects who received the relevant story analogy, i.e., the dispersion solution was most frequent for the Attack-Dispersion condition, the open passage solution was most frequent for the Open Supply Route condition, and operation solutions were most frequent for the Tunnel condition.

TABLE 13.3
Percentage of Subjects in Each Condition of Experiment I Who Proposed Various Solutions to the Radiation Problem

	Proposed solution		
Condition	*Dispersion*	*Open Passage*	*Operation*
Attack-Dispersion story	100	10	30
Open Supply Route story	10	70	50
Tunnel story	20	30	80
Control	0	20	50

These differences in solution frequencies were most dramatic for the dispersion solution. All 10 subjects who were given the Attack-Dispe-rsian story produced this sulution whereas not a single control subject did so. ~"~trr order" ΐ(Γ"δTΰ5ΐη TTevidence regarding a possible set effect, the frequencies of specific radiation solutions, other than those analogous to the various stories, were tabulated for each condition. The solutions examined were proposals to treat the healthy tissue directly, rather than altering the route that the rays take. Specifically, these solutions suggested decreasing the sensitivity of the healthy tissue to the rays (e.g., by a chemical injection, or building up a tolerance to the rays), or covering the healthy tissue with a barrier to protect it from the rays (e.g., by inserting a lead shield to protect the healthy tissue). Such solutions were produced by 30% of the subjects in the Control condition, 10% of the subjects in the Tunnel condition, and none of the subjects in the Attack-Dispersion and Open Supply conditions. While the numbers involved were too small to be statistically reliable, these results suggest that an analogy may tend to block generation of alternative types of solutions.

[...] The results discussed so far demonstrate that story analogies can play a major role in directing the problem-solving process. However, they reveal little about the process by which subjects arrive at an analogous solution. We therefore supplemented the quantitative analysis of solution types, ... with a more qualitative analysis of subjects' problem-solving, protocols. Several aspects of the protocols were examined. Occasions when the experimenter prompted the subjects to use the story were noted, as were correspondences between the story and the target problem that subjects mentioned in the course of generating solutions. This analysis was, of course, constrained by the overall quality and quantity of the protocols. For example, some subjects insisted that talking aloud hindered their thinking, and consequently did not say very much. Rather than presenting an exhaustive analysis of all the protocols, we will therefore concentrate on particularly suggestive excerpts. While this type of protocol analysis has obvious limitations, it may at least provide some hints about the process of analogical problem solving, and in fact served in part to motivate subsequent experiments.

A major issue, raised earlier, concerns the degree of mapping subjects perform in the process of generating an analogous solution. Do subjects make use of detailed correspondences between the story and the target problem, or do they focus directly on the solution embedded in the story and attempt to apply it to the target problem? Of the 10 subjects in the Attack-Dispersion condition, 7 produced the dispersion solution without any prompt, and 3 produced it after being prompted to refer back to the story. In some respects the protocols for prompted subjects are potentially more informative, since what such subjects say is more likely to reflect an ongoing solution process, rather than the result of a process already completed. The protocols of 2 of the 3 prompted subjects suggested use of a solution-focusing strategy.

Table [13.4] presents an excerpt from the protocol of one of these subjects, S15. After the prompt to use the story, this subject clearly focuses on the solution of dividing up the army into groups and immediately generates the parallel solution to the radiation problem. There is no apparent mapping between the initial problem stated in the story and the target problem.

Notice also that the solution S15 proposes prior to the prompt involves the idea of applying many low-intensity rays. After the prompt, the subject produces the dispersion solution by augmenting this aspect of the earlier solution with the idea of sending rays from many angles. This pattern of gradual solution development was also evident in the protocol of another prompted subject in the Attack-Dispersion condition.

[…]

The details of the problem-solving process are less evident in the protocols of the seven unprompted subjects, since they expressed the solution all at once. Three of these subjects simply stated the solution and alluded to the usefulness of the prior story (saying, for example, "considering the problem before"). These subjects did not mention any specific correspondences between the story and the target problem, anil hence their protocols were quite unrevealing with respect to the solution process.

However, two other unprompted subjects did spontaneously mention correspondences between the problems. Immediately after reading the radiation problem, S23 stated:

> Like in the first problem, the impenetrable fortress, the guy had put bombs all around, and the bombs could be compared to the destruction of healthy tissue. And so they had to, they couldn't go in in mass through one road, they had to split up so as not to destroy the healthy tissue. Because if there's only a

TABLE 13.4
Portion of Protocol for SI 5 (Attack-Dispersion Condition)

Subject reads radiation problem,
S: Alright I, what I most, what I'd probably do is send in the ray at sufficiently high intensity and then taking the risk that the tissues, the healthy tissues that would be destroyed, could be repaired later on. Trying to relate this to the other problem. I could say that you could give multiple treatments of low-intensity ray. But from this problem it seems that they won't have effect on the tumor so ... so I don't think that would work.
Later...
E; Okay. And as a last question can you give me a, tell me ways in which your solution would satisfy the constraints of the experiment?
S: What are the constraints of the experiment?
E; Okay, i.e.. that the healthy tissue will not be destroyed, and the tumor will be?
S: Alright, in that way my first suggestion would probably not be the way to go at it. Because that way you're getting low intensity so it won't destroy the tissue and • hopefully over a period of time the additive effect of low-intensity rays would kill the tumor. But from reading the article, I don't know if that would work or not, because it says that a low-intensity ray doesn't have any effect on the tumor at all. So I don't know. I don't know any other possible ways of doing it.
E: Would it help to possibly go back to the story and see whether you can apply that?
S: Well, that's what I was trying to do here. It says here he divides his army into different small groups. Okay, may ... possibly. What they could do, but this is a whole new solution now, possibly what they could do is attack the tumor from a multiple of directions with lower intensity rays and then, since you're coming in from all different directions, the healthy, with small-intensity rays you're not going to be destroying the healthy tissue but you're, and they'll all converge at the point of the tumor which will hopefully destroy the tumor.

little bit of ray it doesn't damage the tissue, but it's all focused on the same spot.

[...]

It is clear in the above two cases that the subjects noticed some correspondences involving the initial conditions and constraints of the story and target problem. However, it is difficult to tell whether these aspects of the mapping process were instrumental in generating the analogous solution, or whether subjects simply mentioned the correspondences to justify the adequacy of the solution, after it had already been generated. In general it was not clear what particular correspondences were central to the solution process. However, several subjects alluded to the importance of the phrase "like spokes on a wheel." Recall that the existence of multiple routes is a critical enabling condition for the solution embodied in the Attack-Dispersion story, and it has no explicit parallelin the statement of the radiation problem. This aspect of the story analogy may therefore serve to generate the critical insight that it is possible to send rays from multiple directions. One illustrative example is the following excerpt from the protocol ol'S7 in the Attack-Dispersion condition, which begins immediately after the subject had read the radiation problem:

> Well, I already know the answer. I knew it when I read it. I might as well stop and say that. What you do is you have a bunch of rays that are weaker, and you point them so that they concentrate on one point. So you just have many different angles. It could not only be two dimensional, the analogy of the spokes on the fortress. But you could even have it three dimensional, so you could have a whole ball of spokes going in. And you would have a high intensity right at the tumor.

In addition, the protocols of all three subjects in the Open Supply Route and Tunnel conditions who produced the dispersion solution suggested that it was triggered by the idea of multiple converging routes. For example, immediately after S2 in the Open Supply Route condition read the problem, she expressed the idea of using a "circular approach" (which in her earlier story summary she explicitly related to the phrase "spokes on a wheel"). This idea then led to the multidirectional aspect of the dispersion solution to the radiation problem.

Two subjects in the Tunnel condition who produced the dispersion solution did so after first spontaneously remarking that the general might have sent his men down multiple roads. One other subject, in the Open Supply Route condition, also suggested the dispersion solution to the military problem, but failed to apply it to the radiation problem. [. ...]

Subjects' protocols also provided information about some of the difficulties they encountered in attempting to apply analogies based on the Open Supply Route and Tunnel stories. As we pointed out earlier, the open passage solution is not especially practical and some subjects may have thought of this solution without mentioning it. For example, S32 in the Tunnel condition gave the open passage solution as an afterthought at the very end of the interview, and also outlined the problems with it: that the esophagus is not straight, and that there would be "refraction off the esophagal walls, or absorption of the rays," which would destroy tissue. Three subjects attempted to overcome this difficulty by suggesting that a ray-proof tube through which the rays could be directed should be inserted down the throat.

In addition, the nature of the analogy suggested by the Open Supply Route story is somewhat different from that suggested by the other two stories. The solutions embodied in both of the latter stories suggest procedures that can be used to generate parallel solutions to the radiation problem (dividing the rays in the case of the Attack-Dispersion story, operating in the case of the Tunnel story). In contrast, the Open Supply

Route story only suggests that an existing open passage might be used. The subject must then search his memory to find a concrete example of such an open passage to the stomach (e.g., the esophagus). Applying the analogy thus involves two steps: mapping the abstract idea of an open passage from the story to the target problem, and then thinking of a concrete example of such a passage. The difficulty of applying the analogy may account for the fact that four of the seven subjects in the Open Supply Route condition who gave the open passage solution had to be prompted to use the story.

Table [13.5] presents a portion of the protocol for S19 in the Open Supply Route condition. This subject works through a rather detailed mapping of the correspondences between the story and the radiation problem. But while she clearly develops the abstract idea of finding an open passage, she fails in the attempt to think of a concrete example. The partial solution produced by S19 can be contrasted with the complete lack of success apparent in the protocol of S37 in the Open Supply Route condition:

> The only rhing that is apparent to me is that the general had other information that he knew that one of the thoroughfares would be left open, and so he was able to use that. But, unless the doctor had some new information or some other treatment, I don't see any other applications from the first problem to the second problem.

TABLE 13.5
Portion of Protocol for S19 (Open Supply Route **Condition)**

E: It might help if you reread the instructions here. This part.

(S rereads radiation problem.)

S: Okay, so what was the first problem? The spokes of the wheel—right?
E: Right.
S: So the center fortress deal would be the idea of the tumor. That's...
E: Okay.
S: And then the spokes that blow up a little would be like the healthy tissue that blows up a little bit. And so with that one the guy had one route that was gonna do it. I guess in this one that's what you have to do is find the one route that would work.
E: Okay.
S:. And, I think, and not use the otherways.
E: Okay. What would that be?
S: That would mean we have to find one approach that was going to get to the tumor without getting the healthy tissue. And I don't see how you could do that. Cause it's not—it doesn't seem like it's the same thing.
E: What doesn't seem like the same thing?
S: Well the idea that road, with a road its possible to have one road unguarded but without, in your body there's always going to be, unless the tumor was right on the outside, there would always be some tissue you would have to go through to get tt it.

TABLE 13.6
Portion of Protocol for S24 (Tunnel Condition)

E: If you read your instructions, it says that this story might give you some hints ... What are you thinking?
S: Well, I remember that the main way they solved this problem was they dug under the fortress and went around it. So, possibly in this situation, you could go around the healthy tissue. I don't know how you'd do that... I see an analogy, it's not real clear.
E: Why isn't it clear?
S: Because when I picture healthy tissue in my mind, healthy tissue all over and, you know, just like a tumor in between all this healthy tissue. But here, the mines they're on top near the surface of the ground, so they can, you can dig under those and you won't really have any problem. But here no matter where you go, like a circle around the healthy tissue ... maybe an operation.
E: Except that one of the constraints of the experiment says that you can't operate.
S: Okay, that's not possible ... maybe ... I was thinking that maybe you could give intervals of high intensity, but I don't know, that still would probably destroy the healthy tissue.
E: Can you think of anything else? ... Is this problem, the previous story, is that distracting?
S: (mumbles) Again, I'm looking for an analogy between the two. And kind of set up the same problem and solve it the same way, but I don't know if I can or not.
E: So, can you think of any other possibilities?
S: (long pause)... No.

Notice that S37 appears to have mapped the story and the target problem at a very abstract level of macrostructure, so that the perceived analogy (the general had new information, so perhaps the doctor might also) was too vague to yield a specific solution proposal for the radiation problem.

In the case of the Tunnel condition four of the eight subjects who generated operation solutions received a prompt to use the story before they did so. Some subjects may have been reluctant to suggest an operation solution because they were aware that it violated a constraint given in the problem statement. An excerpt from the protocol of S24, presented in Table [13.6], illustrates the kind of difficulty encountered in this condition. The protocol suggests that the subject was quite carefully mapping components of the story onto components of the radiation problem. However, the subject was unable to generate a satisfying parallel solution to the target problem.

The overall impression created by the problem-solving protocols is that the generation of analogous solutions involves a conscious process of mapping correspondences between the story and the target problem. The degree of mapping required seems to vary a great deal. Sometimes mapping was done in considerable detail, particularly if the subject was having difficulty producing a parallel solution. In other cases noticing one or two major points of correspondence seemed sufficient to generate the solution. In some instances, particularly for dispersion and open passage solutions, aspects of the solution were clearly generated in sequential steps.

Experiment II was designed to provide additional information about the degree of mapping required to produce a solution on the basis of story analogy.

EXPERIMENT II

For Experiment II, a new story, the Parade-Dispersion story was generated. This story retained the critical enabling conditions for the dispersion solution (centrally located fortress, multiple roads radiating outward), but in other ways was substantially disanalogous to the radiation problem. (Table [13.7] presents a schematic outline of the Parade-Dispersion story). In the parade story, the general is not trying to attack the dictator in the fortress, but rather to stage a parade that meets the dictator's specifications. The constraint of the mined roads has been removed. In the Parade story, the procession of soldiers to the fortress directly constitutes the goal state, whereas in the attack story the similar movement of troops is simply the means by which the final goal (capturing the fortress) can be achieved. Thus, even though the surface description of the solution is the same in both stories, the solution contexts differ.

TABLE 13.7
Schematic Outline of Parade-Dispersion Story

Problem statement	
Problem setting	General has army. Country has dictator. Dictator in fortress in center of country, surrounded by villages. Roads radiate from fortress like spokes on a wheel.
Desired goal	Produce impressive parade that can be seen and heard throughout entire country.
Problem constraints	Sending entire army down one road fails to produce impressive parade. If parade fails to impress dictator, genera] will lose his rank.
Solution	
Dispersion	Divide up parade so that each part of country sees part of parade. Use (1) Many small groups of men (2) From different directions (3) Simultaneously
Resulting goal state	Parade seen and heard simultaneously throughout country. General preserves his rank.

TABLE 13.8
Percentage of Subjects in Each Condition of Experiment II Who Proposed the Dispersion Solution to the Radiation Problem

	Dispersion solution			
Condition	*Complete*	*Partial*	*Total*	·
Attack-Dispersion story	57	19	76	47
Parade-Dispersion Story	31	18	4У	45
Control	8	0	К	50

In Experiment II, subjects were divided into 3 conditions. In the two experimental conditions, subjects received either the Attack story or the Parade story, and were then asked for possible solutions to the radiation problem having been told that the [preliminary] story might contain some hints for solving the problem. In the control condition, no initial story was given, subjects being asked simply to solve the problem in the radiation story. The results are summarized in Table [13.8].

[. . .JUnlike subjects in Experiment I, subjects in Experiment II were not prompted to fully explicate their solutions.· As a result, a number of. subjects produced incomplete versions of the dispersion solution. In order to be scored as a complete dispersion solution, three features had to be present in the proposal: (1) the rays are applied to the tumor from different directions, (2) at low intensity, and (3) simultaneously. A partial solution had to contain at least the first feature, the critical element of dispersion. However, partial solutions might omit features 2 and/or 3. [...]

The basic results of Experiment II are ... extremely clear. First, subjects can readily use story analogies to guide their attempts to solve the radiation problem, even without feedback from the experimenter. Second, the effectiveness of analogies in prompting a specific solution is a matter of degree. In particular, a story with a problem statement analogous to that of the radiation problem (the Attack story) was more likely to trigger the dispersion solution than was a story with a problem statement less related to that of the radiation problem (the Parade story). This was true even though both stories embodied similar setting information and solution statements.

Our central concern in the experiments reported so far was to determine if people can use an analogy to generate a solution to a target problem, and to investigate how analogical problem solving proceeds. Consequently, we simplified the subjects' task in several important ways. First, subjects were always allowed to reread the story analogy at any time, so that their performance would not be limited by memory factors. Second, the story was always presented alone, so that subjects would have no problem identifying the relevant analogy. Third, subjects were always explicitly told to use the story as an aid in solving the target problem. This hint was quite nonspecific; at no time were subjects told the nature of the analogous relationship between the story and problem. Nevertheless, the hint eliminated the need for subjects to spontaneously notice the possible analogy.

In many cases of everyday problem solving in which an analogy could help, the person would have to spontaneously notice the correspondence between the target problem and some analogous problem, either of which might be stored in memory. The two experiments reported below begin to investigate the effect of such additional processing requirements on analogical problem solving.

EXPERIMENT [III]

In Experiment [III], two 'distractor stories' were given to all subjects along with the Attack-Dispersion story. These stories were intended to be as disanalogous to the radiation problem as possible while being matched for length and maintaining the basic problem-with-solution format.

The critical point in Experiment [III] was that while all subjects were given the radiation problem to solve after having studied the three preliminary problem stories, only half the subjects were given the instruction that One of the stories you read will give you a hint for a solution to this problem'.

The results of this experiment were striking. […] Whereas 92% of the subjects in the Hint condition produced the dispersion solution, only 20% (3 out of 15) of those in the No Hint condition did so. … Furthermore, 2 of these 3 subjects gave only partial solutions (as defined in Experiment II), and indicated that they did not consider using the stories. It is therefore possible, and in fact rather likely, that only 1 of the 15 subjects spontaneously noticed the critical analogy and successfully applied it to produce the dispersion solution.

EXPERIMENT [IV]

The results of Experiment [III] demonstrated that subjects can identify a relevant story analogy encoded into memory in the context of distractor stories, and can use the analogy to generate a solution to a subsequent target problem. However, when the experimental instructions did not provide a hint that the stories might help to solve the target problem, subjects seldom noticed or used the analogy. This suggests that the process of analogical problem solving is neither automatic nor invariably applied by college students as a conscious strategy. The knowledge acquired in the context of the story recall phase of the experiment seemed to be encapsulated in such a way that its pertinence to the problem-solving task was not recognized.

An important question is whether this type of encapsulation of experience is more or less absolute, or whether there are factors that would make a relevant analogy more likely to be noticed even though it was initially encoded in a recall context. Experiment [IV] modified the design of Experiment [III] in order to examine two such possible factors. First, the total memory load was reduced by eliminating the two distractor stories from the recall phase; and second, in one condition the story analogy was presented *after* subjects had read and begun to work on the radiation problem. The latter condition can be viewed as an experimental analog of a situation in which a person "stumbles upon" relevant information in the course of working on a problem, as is often reported in anecdotes describing the "Eureka" experience of creative thinkers.

Table [13.9] presents the percentage of subjects in each condition who produced the dispersion solution during the various steps of the procedure. There was no evidence that the manipulation of presenting the problem prior to the story analogy (Story Second condition) increased the probability that subjects would notice or use the analogy. In the Story First condition, 41% of the subjects gave the dispersion solution on their first attempt following recall of the story; while in the Story Second condition, 35% of the subjects produced this solution immediately after reading the story. One subject in the Story First condition gave a partial solution; all other solutions were complete. For the Story First condition we cannot clearly separate subjects who used the story to produce the solution from those who may have produced it spontaneously (as did those subjects in the Story Second condition who gave the dispersion solution prior to seeing the story). However, of the seven subjects in the Story First group who gave the dispersion solution immediately after story recall, six reported that they used the story to help solve the problem. If we accept these reports at

TABLE 13.9
Percentage of Subjects in Experiment [IV] Who Produced Dispersion Solution at Each Step of the Procedure

Condition	*Before story*	*After story (ηυ hint)*	*After story (with hint)*	*Never*	*N*
Story First	—	41	35	24	17
Story Second	10	35	30	25	20

face value, it appears that the percentages of subjects in the two conditions who spontaneously noticed and used the analogy were identical (35% in both conditions). The percentages of all subjects who reported that it occurred to them to use the story were also similar across the two conditions (47% in the Story First condition, 40% in the Story Second condition).

GENERAL DISCUSSION

The present study provides an experimental demonstration that a solution to a problem can be developed by using an analogous problem from a very different domain. Our results substantiate anecdotal descriptions of the role that analogical thinking may play in creative problem solving, and at the same time provide some information about the mental processes involved in analogical problem solving. The results of Experiments I and II indicated that there is considerable variation in the degree of mapping required to generate an analogous solution. In particular, the intermediate frequency of dispersion solutions produced in Experiment II by the Parade story, which was only partially analogous to the radiation problem, supports two important conclusions about the mapping process involved in analogical problem solving. First, subjects in the parade condition were much more likely to generate dispersion solutions than were control subjects. Thus it seems that subjects can often generate an analogous solution even though a complete mapping between aspects of the prior story and the target problem is impossible. In such cases it seems that a solution-focusing strategy may be sufficient to produce the parallel solution. Second, the Parade story was not as effective as the more completely analogous Attack story in prompting the dispersion solution. This suggests that subjects can also perform a more detailed mapping between the problem statements of the story and of the target problem, and that these additional correspondences are sometimes critical in determining whether the subject arrives at the analogous solution.

However, the types of correspondences between the two problem statements that are most critical in developing a solution are not entirely clear. Numerous subjects in our experiments commented on the importance of the reference in the story to roads radiating outward "like spokes on a wheel." Intuitively, this phrase seems to elicit a spatial image that represents those essential aspects of the dispersion solution that can be applied to both the military and the medical problems. Even though the stories and the target problem were always presented verbally in our experiments, the problems essentially describe spatial relationships. Our use of a propositional representation to describe the correspondences between the stories and the radiation problem does not preclude the possibility that some form of analog representation plays an important role in the mapping process. For example, the mapping process may in part depend on interpretive procedures that are applied to a mediating spatial image. Further research is needed to explore the role of spatial representation in analogical problem solving.

It is clear that our understanding of the use of analogies in problem solving remains severely limited in many important respects. We certainly need to be cautious in generalizing on the basis of the present study, which used only one target problem and a very limited set of story analogies. While it seems reasonable to expect that comparable results would be obtained with other ill-defined "insight" problems, for which a solution hinges on a small number of critical inferences, this remains to be demonstrated.

It is still less clear whether analogies can be used in a similar fashion to help solve more "computational" problems, for which the solution consists of a series of discrete steps. Reed et al. (1974) were unable to demonstrate positive transfer between two homomorphic "river crossing" problems, except when the correspondences between the arguments of the two problems were described to subjects. In addition, most subjects in the Reed et al. study reported making little or no use of the first problem when solving the second. It is possible that the mapping process required in such multimove problems places excessive demands on memory capacity. [...] In addition, it is possible that people are able to use analogies more easily in solving some computational problems than in solving others. -For example, Hayes and Simon (1977) have demonstrated positive transfer between isomorphic versions of the Tower of Hanoi puzzle, another computational problem. Clearly much remains to be learned about the influence of problem characteristics on problem solving by analogy. In addition to investigating the effects of problem type, we need to learn more about the ways in which the use of analogies may interact with other strategies (e.g., means-ends analysis) used in problem solving.

Noticing and Accessing Potential Analogies

A number of important questions for future research involve the closely related issues of the spontaneous noticing of analogies, and the accessing of potential analogies stored in memory. The results of Experiments [III] and [IV] suggest that one of the major blocks to successful use of analogy may be failure to spontaneously notice its pertinence to the target problem. When subjects were not told to try to use the prior stories to help solve the radiation problem, only a minority succeeded in generating the analogous solution. This decline in transfer performance cannot be attributed to faulty encoding of the story analogy, since most subjects were able to produce the analogous solution once they were given a hint to apply the story. Also, the problem of spontaneous noticing was not limited to stories previously encoded into memory. In the Story Second condition of Experiment [IV], many subjects failed to notice the relevance of the story even though they had to read, memorize, and recall it *after* beginning to work on the target problem.

[...]

Why should subjects so often fail to notice the relevance of a story analogy to a target problem when a hint to use the story is not provided? One might argue that this result is not particularly surprising, since the story was presented in a different experimental context (a story recall experiment). The difficulty of the recall context may be related to the problem of identifying the optimal level of abstraction for representing an analogy, as we discussed in the Introduction. A recall task, with its emphasis on memory for specific wording, may lead the person to represent the story at a level of macrostructure too detailed to maximize its analogical correspondence with the target problem. A hint to use the story may lead the person to derive a more abstract level of macrostructure, better suited for the problem-solving task.

But in uny case, the issue of how analogies are noticed is a very general one. A potential analogy may often be encoded in a very different context from that in which the target problem appears. Indeed, the basic problem in using an analogy between remote domains is to connect two bodies of information from disparate semantic contexts. More generally, successful transfer of learning generally involves overcoming contextual barriers. This may not be easy; for example, it is all too common for a student to fail to notice the relevance of knowledge acquired in one class to a problem encountered in another.

The problem of how analogies are noticed is closely related to the issue of how analogies are accessed in memory. Noticing that information in memory is relevant to a target problem is part of the process of retrieving an analogy. These problems were side-stepped in Experiments I-II, since subjects- received a hint to use the story analogies and were allowed to reread them at any time. The problem of memory access was greatest in Experiment [III], in which the relevant story analogy was memorized in the context of two irrelevant distractor stories. Subjects in this experiment seemed to have little difficulty in identifying the appropriate story in memory, and applying it to the target problem, as long as they were instructed to do so. However, subjects may have performed this task by simply testing each of the three stories to see if it suggested a solution to the target problem. Such a strategy would presumably be impractical in most everyday problem-solving situations, where virtually any piece of information in memory might potentially afford a useful analogy.

How might potential analogies be accessed in memory? Is the memory search process directed, and if so, how? At one extreme the problem solver may not actually search memory at all; rather, he or she may simply "stumble upon" an analogy. That is, after a piece of knowledge has for some reason become the focus of attention, the person may spontaneously notice its analogous relationship to a problem yet to be solved. It also seems plausible, however, that people may sometimes locate useful analogies in memory on the basis of a conscious search process. It may be possible to use a representation of the current problem as a retrieval cue for accessing analogous problems. Perhaps in some cases the person first begins working on a problem and arrives at an abstract characterization of a potential solution, as we discussed in Experiment I. This solution representation might then be used to retrieve an analogous problem with that type of solution, which could then be used to help generate a more concrete solution to the target problem. The latter possibility is related to the solution-focusing strategy discussed in connection with Experiment I. A better understanding of how analogies are retrieved and noticed is clearly essential in order to effectively teach the use of analogies as a heuristic strategy for problem solving.

The Generality of the Mapping process

The mapping process involved in the use of analogies may play a role in a variety of cognitive skills. Using an analogy involves mapping.the representations of two (or perhaps more) instances onto one another. Similar processes may also be involved in *abstracting* the relational structure common to a set of particular instances. In the domain of problem solving, for example, a person who encounters several analogies to the radiation problem might eventually derive a schema for "dispersion-type" problems. This schema would presumably be structured much like a concrete "instance of a dispersion problem (cf. Figure [13.1]), except that the predicates and arguments would be more abstract. A person equipped with such a general schema could then solve new dispersion-type problems by mapping them directly onto it. These observations suggest that similar mapping processes may be involved in three distinct but interrelated activities: (1) comparing one instance to another; (2) deriving a schema for a class of instances; and (3) comparing an instance to a general schema.

Note that the above description of the role of mapping potentially applies not just to problem solving, but to a wide range of cognitive skills requiring concept learning and classification of instances. Such skills are involved in tasks that vary a great deal in terms of both complexity and cognitive domain. For example, the mapping of correspondences between relational structures is involved in the use of schemata for story understanding (Rumelhart, 1975), frames for scene perception (Minsky, 1975), and scripts for understanding of social behavior (Abelson, 1975). Such structures all serve to describe our ability to deal with novel instances of familiar situations. Theories in each domain must explain how abstract structures can be derived from a set of instances, and how instances can be related to each other and to abstract structures.

If similar mapping processes are involved in analogical problem solving and other cognitive skills, then the study of the use of analogies to solve problems has implications that extend to other domains. We mentioned at the beginning of this paper that an analogy may often serve as a model to guide the development of a new theory. In a similar fashion a theory of analogical problem solving might serve as a useful model in developing theories in other areas of cognition.

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